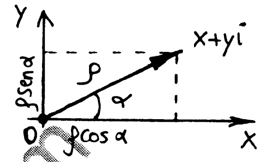
Números Complejos

Binómica: $z = x + yi \quad x, y \in \mathbb{R}$ Modulo $z = \rho = |z| = \sqrt{x^2+y^2}$

Argumento de $z = \arg z = \operatorname{arctg} y/x = \alpha$

Trigonométrica: $z = \rho\cos\alpha + i\rho\operatorname{sen}\alpha = \rho(\cos\alpha + i\operatorname{sen}\alpha)$



Fórmula de Euler: $e^{\alpha i} = \cos\alpha + i\operatorname{sen}\alpha$

Exponencial: $z = \rho e^{\alpha i}$ $\qquad -z = \rho e^{(\pi+\alpha)i}$

Suma de complejos: $\left.\begin{array}{l} z_1 = x_1 + iy_1 \\ z_2 = x_2 + iy_2 \end{array}\right\} z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$

Producto de complejos: $\left.\begin{array}{l} z_1 = x_1 + iy_1 \\ z_2 = x_2 + iy_2 \end{array}\right\} z_1 \cdot z_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1)$

$\left.\begin{array}{l} z_1 = \rho_1 e^{\alpha_1 i} \\ z_2 = \rho_2 e^{\alpha_2 i} \end{array}\right\} z_1 z_2 = \rho_1\rho_2 \cdot e^{(\alpha_1+\alpha_2)i}$

Cociente de complejos: $\left.\begin{array}{l} z_1 = x_1 + iy_1 \\ z_2 = x_2 + iy_2 \end{array}\right\} \dfrac{z_1}{z_2} = \dfrac{x_1+iy_1}{x_2+iy_2} \cdot \dfrac{x_2-iy_2}{x_2-iy_2} = \dfrac{(x_1x_2+y_1y_2)+i(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$

$\left.\begin{array}{l} z_1 = \rho_1 e^{\alpha_1 i} \\ z_2 = \rho_2 e^{\alpha_2 i} \end{array}\right\} z_1/z_2 = \dfrac{\rho_1}{\rho_2} e^{(\alpha_1-\alpha_2)i}$;

Complejo conjugado:
$z = a+bi \rightarrow \bar{z} = a-bi$
$z = \rho e^{\alpha i} \rightarrow \bar{z} = \rho e^{-\alpha i}$

Potencia de exponente entero: $(\rho(\cos\alpha + i\operatorname{sen}\alpha))^n = \rho^n(\cos n\alpha + i\operatorname{sen} n\alpha) = \rho^n e^{n\alpha i}$

Raíces n-ésimas de números complejos: $\sqrt[n]{\rho e^{\alpha i}} = \sqrt[n]{\rho}\cdot e^{\frac{\alpha+2k\pi}{n}i} \qquad k = 0, 1, 2, \ldots, (n-1)$

Logaritmo neperiano: $\ln(x+yi) = \ln(\rho e^{\alpha i}) = \ln\rho + (\alpha + 2k\pi)i \qquad k \in \mathbb{Z}$ $\qquad \log_a b = \dfrac{\ln b}{\ln a}$

Principal si $k = 0$; $\ln(x+yi) = \ln\rho + \alpha i$

Potencias de base y exponente complejo: $z^w = (a+bi)^{c+di} = e^{(c+di)\cdot\ln(a+bi)}$

Funciones trigonometricas: $e^{\alpha i} = \cos\alpha + i\operatorname{sen}\alpha \qquad e^{-\alpha i} = \cos\alpha - i\operatorname{sen}\alpha$

$\cos z = \dfrac{e^{zi} + e^{-zi}}{2} \qquad \operatorname{sen} z = \dfrac{e^{zi} - e^{-zi}}{2i}$

Funciones hiperbólicas: $\operatorname{Ch} z = \dfrac{e^z + e^{-z}}{2} \qquad \operatorname{Sh} z = \dfrac{e^z - e^{-z}}{2}$

Relaciones: $\operatorname{Ch}^2 z - \operatorname{Sh}^2 z = 1 \qquad \cos z = \operatorname{Ch} iz \qquad \operatorname{Ch} z = \cos iz$

$\operatorname{Ch}^2 z + \operatorname{Sh}^2 z = \operatorname{Ch} 2z \qquad \operatorname{sen} z = \dfrac{1}{i}\operatorname{Sh} iz \qquad \operatorname{Sh} z = \dfrac{1}{i}\operatorname{sen} iz$