TRANSFORMADAS DE LAPLACE

$$\mathcal{L}[f(x)] = \int_0^\infty e^{-sx} f(x)\,dx$$

LINEALIDAD: c_1 y $c_2 \in \mathbb{R}$

$$\mathcal{L}[c_1 f_1(x) + c_2 f_2(x)] = c_1 F_1(s) + c_2 F_2(s)$$

con $F_1(s) = \mathcal{L}[f_1(x)]$

$F_2(s) = \mathcal{L}[f_2(x)]$

* $\mathcal{L}[e^{ax} f(x)] = F(s-a)$
* $\mathcal{L}[x^n f(x)] = (-1)^n \frac{d^n}{ds^n} F(s)$
* $\mathcal{L}\left[\frac{f(x)}{x}\right] = \int_s^\infty F(t)\,dt$ si existe el $\lim_{x\to 0^+} \frac{f(x)}{x}$
* $\mathcal{L}[f'(x)] = s\,\mathcal{L}[f(x)] - f(0)$
* $\mathcal{L}\left[\int_0^x f(t)\,dt\right] = \frac{1}{s} F(s)$

FUNCIONES PERIODICAS: Si $f(x)$ es periodica de periodo T.

$$\mathcal{L}[f(x)] = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\,dt$$

CONVOLUCION:

$$\mathcal{L}[f(x) * g(x)] = \mathcal{L}[f(x)] \cdot \mathcal{L}[g(x)]$$

TRASLACION: $\mathcal{L}[f(x-c)] = e^{-cs}\,\mathcal{L}[f(x)]$ con $c>0$.

Para ecuaciones diferenciales:

$$\mathcal{L}[y'] = sY(s) - y(0) \quad \text{con} \quad Y(s) = \mathcal{L}[y(x)]$$

$$\mathcal{L}[y''] = s^2 Y(s) - sy(0) - y'(0)$$

$$\mathcal{L}[y'''] = s^3 Y(s) - sy(0) - y'(0) - y''(0)$$

TRANSFORMADAS DE LAPLACE	
$f(x)$	$L(f(x))$
1	$\frac{1}{s}$ $s>0$
x^{n-1}	$\frac{(n-1)!}{s^n}$ $s>0, n\geq 1$
$\sqrt{x}$	$\frac{\sqrt{\pi}}{2} s^{-\frac{3}{2}}$ $s>0$
$\frac{1}{\sqrt{x}}$	$\sqrt{\pi}\, s^{-\frac{1}{2}}$ $s>0$
e^{ax}	$\frac{1}{s-a}$ $s>a$
$\operatorname{sen} ax$	$\frac{a}{s^2+a^2}$ $s>0$
$\cos ax$	$\frac{s}{s^2+a^2}$ $s>0$
$x \operatorname{sen} ax$	$\frac{2as}{(s^2+a^2)^2}$ $s>0$
$x \cos ax$	$\frac{s^2-a^2}{(s^2+a^2)^2}$ $s>0$
$x^{n-1} e^{ax}$ $(n\geq 1)$	$\frac{(n-1)!}{(s-a)^n} s>a$
$e^{bx} \operatorname{sen} ax$	$\frac{a}{(s-b)^2+a^2}$ $s>a$
$e^{bx} \cos ax$	$\frac{s-b}{(s-b)^2+a^2}$ $s>b$
$\operatorname{sen} ax - a\,x \cos ax$	$\frac{2a^3}{(s^2+a^2)^2}$ $s>0$