

TRANSFORMADAS DE FOURIER

| | $f(x)$ | $f(\omega) = \mathfrak{F}[f(x)]$ |
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| 1 | $e^{-a^2 x^2}$ | $\frac{\sqrt{\pi}}{a} e^{-\frac{\omega^2}{4a^2}}$ |
| 2 | $e^{-a x }, \quad a > 0$ | $\frac{2a}{a^2 + \omega^2}$ |
| 3 | $\begin{cases} 1 & x < a \\ 0 & x > a \end{cases}, \quad a > 0$ | $\frac{2 \operatorname{sen} a \omega}{\omega}$ |
| 4 | $\frac{1}{x^2 + a^2}, \quad \operatorname{Re} a < 0$ | $\frac{-\pi}{a} e^{a \omega }$ |
| 5 | $\frac{x}{(x^2 + a^2)^2}, \quad \operatorname{Re} a < 0$ | $\frac{i\omega\pi}{2a} e^{a \omega }$ |
| 6 | $x e^{-a x }, \quad a > 0$ | $-4 \frac{i a \omega}{(a^2 + \omega^2)^2}$ |
| 7 | $\begin{cases} \cos ax, & x < \frac{\pi}{2a} \\ 0 & x > \frac{\pi}{2a} \end{cases}$ | $\frac{2a}{a^2 - \omega^2} \cos \frac{\pi\omega}{2a}$ |
| 8 | $\begin{cases} 1 - x , & x < 1 \\ 0 & x > 1 \end{cases}$ | $4 \frac{\operatorname{sen}^2 \frac{\omega}{2}}{\omega^2}$ |
| 9 | $\begin{cases} 0, & x < 0 \\ x^k e^{ax}, & x > 0 \end{cases} \quad \operatorname{Re} a < 0$ $k = 1, 2, \dots$ | $\frac{k!}{(i\omega - a)^{k+1}}$ |
| 10 | $\begin{cases} -x^k e^{ax}, & x < 0 \\ 0, & x > 0 \end{cases} \quad \operatorname{Re} a > 0$ $k = 1, 2, \dots$ | $\frac{k!}{(i\omega - a)^{k+1}}$ |
| 11 | $\delta(t - c)$ | $e^{-i c \omega}$ |
| 12 | $\delta^{(n)}(t - c)$ | $(i\omega)^n e^{-i c \omega}$ |
| 13 | 1 | $2\pi \delta(\omega)$ |
| 14 | t^n | $2\pi i^n \delta^{(n)}(\omega)$ |
| 15 | $e^{i a t}$ | $2\pi \delta(\omega - a)$ |
| 16 | $t^n e^{i a t}$ | $2\pi i^n \delta^{(n)}(\omega - a)$ |

| | | |
|----|-----------------------|--|
| 17 | $U(t-c)$ | $\frac{e^{-ic\omega}}{i\omega} + \pi\delta(\omega)$ |
| 18 | $t^n U(t)$ | $\frac{n!}{(i\omega)^{n+1}} + \pi i^n \delta^{(n)}(\omega)$ |
| 19 | $\frac{1}{(t-c)^n}$ | $\frac{(-i\omega)^{n-1} e^{-ic\omega}}{(n-1)!} \pi i [1 - 2U(\omega)]$ |
| 20 | $e^{iat} t^n U(t)$ | $\frac{n!}{[i(\omega-a)]^{n+1}} + \pi i^n \delta^{(n)}(\omega-a)$ |
| 21 | $\frac{e^{iat}}{t-c}$ | $e^{-ic(\omega-a)} \pi i [1 - 2U(\omega-a)]$ |

Representación integral de $f(x)$:

$$f(x) = \int_0^\infty (a(\omega) \cos \omega x + b(\omega) \sin \omega x) d\omega \quad \begin{cases} a(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(u) \cos \omega u du \\ b(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(u) \sin \omega u du \end{cases}$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega x} \left(\int_{-\infty}^{+\infty} f(u) \cdot e^{-i\omega u} du \right) d\omega$$

TRANSFORMADA DE FOURIER:

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx = \mathcal{F}[f(x)]$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) \cdot e^{i\omega x} d\omega$$

$$\mathcal{F}[\alpha f(x) + \beta g(x)] = \alpha \mathcal{F}[f(x)] + \beta \mathcal{F}[g(x)]$$

$$\mathcal{F}[f(x-h)] = e^{-i\omega h} \cdot \mathcal{F}[f(x)]$$

$$\mathcal{F}[e^{ihx} \cdot f(x)] = F(\omega-h)$$

$$\mathcal{F}[f'(x)] = (i\omega) \mathcal{F}[f(x)] \quad \rightsquigarrow \quad \mathcal{F}[f^{(k)}(x)] = (i\omega)^k \mathcal{F}[f(x)]$$

$$\mathcal{F}[f_1(x) * f_2(x)] = \mathcal{F}[f_1(x)] \cdot \mathcal{F}[f_2(x)] \quad (\text{Convolution})$$

Propiedades

$$\mathcal{F}[\delta(t-c)] = e^{-i\omega c}$$

$$\mathcal{F}[\delta^{(n)}(t-c)] = (i\omega)^n e^{-i\omega c}$$

$$\delta^{(n)}(t-c) * f(t) = f^{(n)}(t-c)$$

$$\delta^{(n_1)}(t-c_1) * \delta^{(n_2)}(t-c_2) = \delta^{(n_1+n_2)}(t-c_1-c_2)$$

$$\text{Si } \mathcal{F}[f(t)] = F(\omega) \Rightarrow$$

$$\mathcal{F}[F(\omega)] = 2\pi f(-\omega)$$

$$\mathcal{F}[e^{-at} u(t)] = \frac{1}{i\omega + a} \quad a > 0$$

$$\mathcal{F}[e^{at} u(t)] = \frac{-1}{i\omega - a} \quad a > 0$$